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# Three dimensional image reconstruction algorithm based on ray casting and cloud computing

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**Abstract.** An improved volume algorithm based on spatial tetrahedral subdivision is proposed. The basic principle of tetrahedral subdivision is: the space geometry model is divided into a plurality of tetrahedron units, which constitute convex hulls of target partitions in accordance with certainly topological structure and calculate the volume of models to be tested by calculating all tetrahedral volumes. Excessive packaging detection system is proposed in this paper, firstly, calculate the volume of cloud bounding box in accordance with the ray casting system to obtain 3D surface point cloud of the object, and the object point cloud to obtain the bounding box of point cloud, and then calculate the volume of point cloud through the improved spatial triangulation subdivision method, and finally the volume of point cloud is substitute into the calculation of the plot ratio and void ratio to realize the packaging inspection of commodity. The experimental results show that the accuracy of the detection system meets the relevant requirements, and can effectively improve the measurement efficiency.

**Key words.** Packaging detection, 3D reconstruction, Ray casting, Point cloud computing, Volume measurement.

## 1. Introduction

The traditional method[1] of excessive packaging detection is to manually measure the volume of commodity packaging by vernier caliper and steel ruler, and to achieve the detection by calculating the void ratio[2]. In the face of the increasingly frequent detection process, the demand for packaging detecting instrument with high speed and high precision is gradually increasing. The study[3,4] of relevant technol-

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ogy has gained certain progress, such as small packaging of cigarette online detection technology[3], but the formation of specialized excessive packaging detection instruments program has not been reached, so developing the excessive packaging detection instruments instead of traditional detection methods is a problem to be solved.

In this paper, a method of calculating machine vision measurement system for volume measurement of irregular shaped material stack is presented in literature [4], in the measurement system, the default vertical scale direction is used as the spindle of material stack subdivision. In which, according to the sectional size of the object to be measured to give the size of the reference differential, when the object to be measured is changed, the given differential unit datum may need to be changed, which can not meet the requirement of the system's rapidity. In this paper, a volume algorithm for arbitrary triangular mesh model is proposed in literature [5]. By using the triangle patch in model that is formed by cutting method to specify the projection plane of current mesh, introducing volume concept with the symbol (volume can be positive or negative), and defining each triangle patch and form a new volume of convex pentahedron by specifying projection on plane to calculate the total volume of all convex pentahedrons that is the volume of mesh model, this method belongs to the projection method, which is suitable for the measurement of large volume model with low accuracy, strong dependency to environment and bad feasibility.

## 2. Ray casting measuring system

### 2.1. System solution

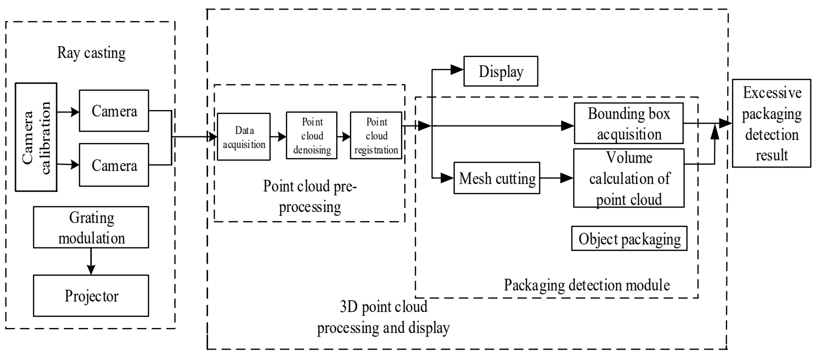



Fig. 1. System functional block diagram

According to the relevant requirements of excessive packaging detection, the design of packaging detection system solution and system function diagram is shown in Figure 1, project the ray fringe image that is modulated in advance on the object surface, the projection information of fringe image on object surface is collected by the camera, according to the changes of fringe pixel value of grating image to convert the 3D data of object surface. The four-step phase-shifting method is used to obtain the 3D point cloud of object. The obtained point cloud data are registered in the same coordinate, and are fused to form the 3D point cloud of object according to the classical registration method. On this basis, to analyze the geometric characteristics of object, conduct the three-dimensional Delaunay tetrahedron subdivision, establish the tetrahedral mesh and calculate the volume of point cloud. The system can analyze the whole shape feature of object, and intuitively give the measurement result of the point cloud to be measured to achieve the real-time measurement of object.

Table 1. Limited requirements of excessive packaging detection

| Product category | Limited index | |
|---|---|---|
| | Void ratio of packaging | Layer of packaging |
| Alcoholic drink | <= 55% | 3 layers or less than 3 layers |
| Pastry | <= 60% | 3 layers or less than 3 layers |
| Grain[a] | <= 10% | 3 layers or less than 3 layers |
| Health food | <= 50% | 3 layers or less than 3 layers |
| Cosmetic | <= 50% | 3 layers or less than 3 layers |
| Other food | <= 45% | 3 layers or less than 3 layers |

Note: When all single net contents of the inside products are not greater than 30mL or 30g, the void ratio of packaging should not exceed 75%; when all single net contents of the inside products are greater than 30mL or 30g, and not more than 50mL or 50g, the void ratio of packaging should not exceed 60%.

[a] Grain refers to primary processed products on the basis of raw grain

According to the volume of the bounding box and the point cloud, packaging detection calculates the plot ratio and void ratio of good. In accordance with the relevant regulations [2], the formula for calculating the void ratio of good is shown in formula (1).

$$X = \frac{[V_n - (1+k)V_0]}{V} \times 100\% \,, \tag{1}$$

$X$ refers to the void ratio of packaging, $V_0$ refers to initial packaging volume of commodity(that is the circumscribed cuboid minimum volume of initial packaging of commodity); $V_n$ refers to sales packaging volume(that is the circumscribed cuboid minimum volume of sales packaging); $k$ refers to necessary space factor, $k$=0.6 in literature[2].

$$\rho = \frac{V_1}{V_2} \times 100\% \,, \tag{2}$$

In which, $V_1$ refers to the volume of commodity, $V_2$ refers to the sales packaging volume of commodity.

The concrete steps of calculating the packaging void ratio and the plot rate of commodity include: calculation of the volume of sales packaging point cloud and bounding box, the volume of initial packing point cloud and bounding box, which are substituted into formula (1) and (2), and the implementation process as shown in figure 2.

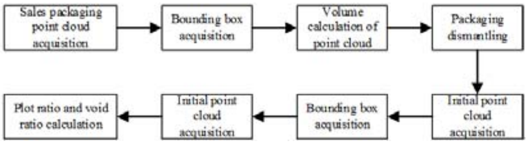



Fig. 2. Packaging detection process

The requirements of the relevant provisions of the food and cosmetic packaging layer and void ratio shall be shown in Table 1:

### 2.2. Principle of measurement

The principle diagram of ray casting measurement is shown in Figure 3, the sinusoidal grating is projected on the measured object surface, the beam offsets due to the height changes of the object, which is modulated by the surface shape, the offset carries the height information of the point, according to the classical triangulation principle, to obtain the height of the point in accordance with the beam deflection. In Figure 3, the grating that is projected on the reference plane is sinusoidal grating, and the object to be measured is put on the reference surface XY (the Y axis is perpendicular to paper), $l$and$d$respectively are the distances between the optical center of the camera to the reference surface, the optical center of projector. See from the principle diagram of optical path, because the height changes of the object, namely by the modulation of surface shape, the beam from point B to the new position point A, the displacement of point B to point A carries the height information $h$of point P.

The phase field that is modulated by object surface is obtained by fringe diagram, the system needs to achieve the processing of grating phase, the common grating phase processing methods include Mohr Method, Fourier Transform Profilometry (FTP), Phase Shift Method and Wavelet Analysis, etc. It is widely used that Fourier Transform Profilometry and Phase Shift Method, and compared with the Fourier Transform Profilometry, Phase Shift Method has better precision, and less computation, less time consuming and good practicality. Phase Shift Method in system is used to process the fringe image, and the phase is used to describe the spatial distribution of the grating field in measurement, and to obtain 3D coordinate of the point through phase position in fringe image. Phase Shift Method is used to increase the number of constant phase by phase shift of grating fringe phase field, and to obtain many pieces of grating fringe image for solving phase field. By analyzing the triangle relation of the system structure, it can be seen that there is a relationship between the height contour $h$of the object and the phase difference $\Delta\theta(m,n)$of the deformation of the grating as shown in formula (3).

$$h = \frac{l(\theta_A - \theta_B)}{(\theta_A - \theta_B) + 2\pi d/\lambda_0}. \tag{3}$$

In the formula, $l$ refers to the distance between the optic center of camera and reference surface, $d$ refers to the distance between the optic center of projector and the optic center of camera, $\lambda_0$ is grating pitch.

According to the optical triangle theory, the optical path shall pay attention to:

(1)The optical axis of projector lens is perpendicular to the reference surface.

(2)The projecting axis of optical axis intersects the reference surface.

(3) The optical axis of fringe image of camera projection is inter-parallel with Y-axis of projector.

According to the height information of the point cloud obtained by above principles, then according to the plane grating fringe to obtain the vertical and horizontal coordinates of each point, the fringe spacing records the horizontal coordinate, and the fringe height records the vertical coordinate, and can obtain the 3D coordinates of the point.

## 3. Volume algorithm

### *3.1. Volume algorithm of bounding box*

The bounding box is a method to find the optimal bounding space of the discrete point set, which is used instead of the circumscribed cuboid to meet the actual demands of the system. As shown in Figure 4, because the uncertainty of the object placement, and each object point cloud has its own geometric characteristics, but the existing coordinate system may not be based on these characteristics, the original coordinate system needs to be converted, the new coordinate system as much as possible to reveal the major changes information of data, the transformation process of the base vector is called "principal component analysis" [6-7]. Aiming at the problem of axial inspection of cuboid and cylinder in packaging detection, the identification of the object's features is realized by the principal component analysis (PCA). Principal component analysis (PCA) algorithm makes use of the idea of dimensionality reduction, to eliminate the interference of redundancy and noise as far as possible to find the so-called "principal component". The principal component analysis is achieved through the Hotelling transform of point cloud coordinates, Hotelling transform is based on statistical distribution to find a set of optimal orthogonal vectors basis to represent the original sample data, and the error of new samples and original sample is small.

The system extracts 3D point cloud coordinate, and applies the Hotelling transform to 3D eigenvector, and then take any point coordinate of point cloud as a 3D vector $X_l = (x, y, z)^T$, to calculate the mean vector of 3D point cloud coordinate, $m_x$ and covariance matrix $C_x$ by vector $X$.

$$m_x = \frac{1}{L} \sum_{l=1}^{L} X_l \,, \tag{4}$$

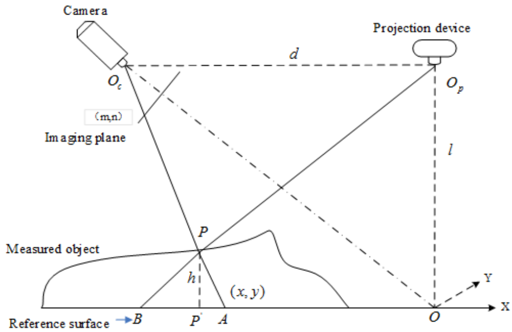
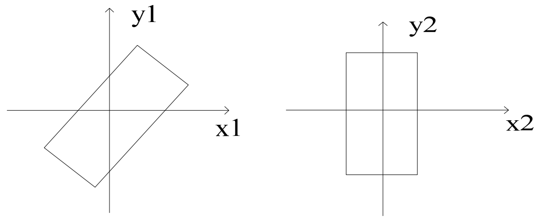



Fig. 3. Optic path diagram of ray casting measuring system




(1) Before transformation (2) After transformation

Fig. 4. Coordinate transformation

$$C_x = \frac{1}{L}\sum_{l=1}^{L} X_l X_l^T - m_x m_x^T \,. \tag{5}$$

In the formula, L refers to the numbers of point in point cloud.

Because $C_x$is 3x3-order real symmetrical matrix, there are 3 eigenvectors, $A = (e_1 e_2 e_3)^T$is the eigenvector matrix of$C_x$, the calculation of point cloud coordinate transformation adopts the Hotelling transform as shown in formula (4). Among which, $A$ is transfer matrix, $X$ is matrix before transformation, $Y$ is matrix after transformation.

$$Y = A(X - m_x)\,, \tag{6}$$

$C_x$is the covariance matrix of $X$, $C_Y$is the covariance matrix of $Y$. Because the row vector of A is the eigenvector of$C_x$, the diagonal element of covariance matrix $C_Y$ of point cloud data in the new coordinate system is the eigenvalue of$C_x$, the corresponding eigenvalue of each vector represents the contribution rate[8] of matrix on this vector, which is shown as formula (8):

$$C_Y = AC_X A^T\,, \tag{7}$$

$$C_Y = \begin{pmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_N \end{pmatrix}. \tag{8}$$

The new coordinate orientation is consistent with the corresponding eigenvector of maximum eigenvalue, and find the maximum and minimal value point on X,Y,Z axis in point cloud data under the new coordinate system, that are $x_{\max}$, $x_{\min}$, $y_{\max}$, $x_{\max}$, $x_{\min}$, $y_{\max}$, which formed the AABB bounding box of object point cloud, to calculate the volume of bounding box of point cloud by formula (9). $V$ refers to the volume of bounding box of point cloud,

$$V = (x_{\max} - x_{\min})(y_{\max} - y_{\min})(z_{\max} - z_{\min})\,. \tag{9}$$

### 3.2. Point-cloud volume computing

The paper is to compute the closed point-cloud volume on the basis of the subdivision of point-cloud tetrahedral mesh. The point cloud will be obtained and spliced by two cameras, so the point cloud is closed. The tetrahedral subdivision of spatial point cloud is based on the algorithm of incremental Delaunays triangulation. The Delaunays optimization criterion of incremental method: in the case of three dimensions, the circumscribed sphere of any tetrahedron does not include other concentrated points. To compute the volume, the tetrahedral mesh subdivision shall be conducted firstly, and the algorithm implementation steps of tetrahedral mesh subdivision are as follows:

(1) Input point cloud data.

(2) Building an initial tetrahedron, and surround the point cloud area to be divided.

(3) Insert point P, and find a tetrahedron T in which the circumscribed sphere includes point P. When a new point P is inserted, following three conditions may occur: ① point P is in T, and T is divided into 4 small tetrahedrons; ② point P is out of T and is located in current tetrahedral mesh. Find the divisional plane between T and P and find the tetrahedron including P. Divide such tetrahedron into 4 small tetrahedrons; ③ point P is out of current tetrahedral mesh. Connect P with 3 peaks (which the planes are visible) of mesh and form a new tetrahedral mesh.

(4) A polyhedron including T is generated by point P and T.

(5) Divide such polyhedron into more than one tetrahedron, and try to avoid such newly generated tetrahedron becomes long and narrow.

(6) If all the points are inserted in current mesh, the peripheral tetrahedron shall be removed, or otherwise, repeat step (3) to (5).

An improved volume algorithm based on Delaunay incremental method is given in the paper. The improvement of volume algorithm is to remove the peripheral tetrahedron. Such peripheral tetrahedron is removed by using the quadric surface fitting point and normal vector of k-nearest point-cloud computing. For any tetrahedron, if there are intersections (excluding such point itself) between each normal vector of current tetrahedron and circumscribed sphere, such tetrahedron is a peripheral tetrahedron, or otherwise is an internal tetrahedron. Remove the redundant tetrahedron in current mesh according to above criterions, and then a relatively accurate point cloud volume will be obtained.

The equation of quadric surface fitting is shown in formula (10):

$$z = ax^2 + by^2 + cxy + dx + ex + f \,. \tag{10}$$

In which $a, b, c, d, e, f$are undetermined parameters.
For the given point of $(x_i, y_i, z_i) i = 1, 2, \cdots, N$, minimize the overall error of $Q$.

$$Q = \sum_{i=1}^{N} [z_i - (ax_i^2 + by_i^2 + cx_i y_i + dx_i + ey_i + f)]^2 \,. \tag{11}$$

In order to solve the undetermined parameters, the extreme value of $Q(a, b, c, d, e, f)$ converted to six-member function can be solved, namely $a, b, c, d, e, f$are able to meet:

$$\frac{\partial Q}{\partial a} = \frac{\partial Q}{\partial b} = \frac{\partial Q}{\partial c} = \frac{\partial Q}{\partial d} = \frac{\partial Q}{\partial e} = \frac{\partial Q}{\partial f} = 0 \,. \tag{12}$$

Compute each undetermined parameter obtained from formula (12), solve the normal vector of such point according to the partial derivative of two-dimension surface equation fitted on any point, and uniform them through normal vector[9].

If the four peaks of such tetrahedron are A$(x_0, y_0, z_0)$, B$(x_1, y_1, z_1)$,C$(x_2, y_2, z_2)$, D$(x_3, y_3, z_3)$, the determinant (13) shall be adopted for the volume V of such tetrahedron.

$$V = \frac{1}{6} \begin{vmatrix} x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} \,. \tag{13}$$

## 4. Experiment and analysis

### *4.1. Experimental platform*

Experimental platform: the projector selected is NEC NP-L51W+; luminance: 500lm; max resolution ratio: 1920×1080; projection distance: 0.52-3m, with the ability of manual zooming and focusing. Measured distance: 500mm-100mm; Max area of object is calculated as 500*500mm. a camera with 10 million pixels is required. The industrial camera (FL2G-13S2C) with 13 million pixels produced by Point Gray is selected. Such camera obtains three-dimensional point cloud at the distance of 60cm away from object. According to relevant standards, the accuracy shall be up to 0.1mm.

### *4.2. Experimental results*

In order to verify the feasibility of detection algorithm of bounding box package, an experiment is conducted by obtaining dense point-cloud data. The obtaining effect of point-cloud bounding box is shown in Fig.5. The first line is original image, and the second line is the results of bounding box detection. It is observed from such results that the Hotelling transform has preferably analyzed the pivot element of point cloud aggregation; basing on the geometrical shape of point cloud, a bounding box is generated; and the effect to detect the package with such bounding box instead of circumscribed rectangular parallelepiped is good.

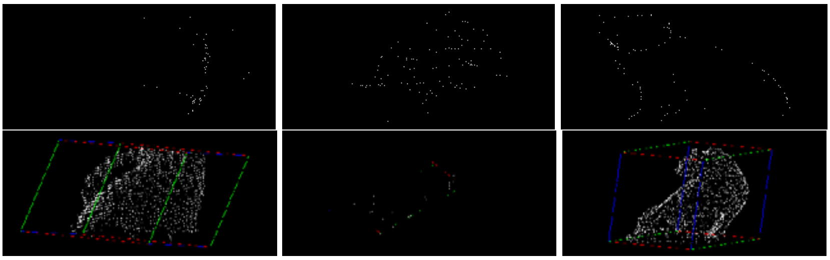
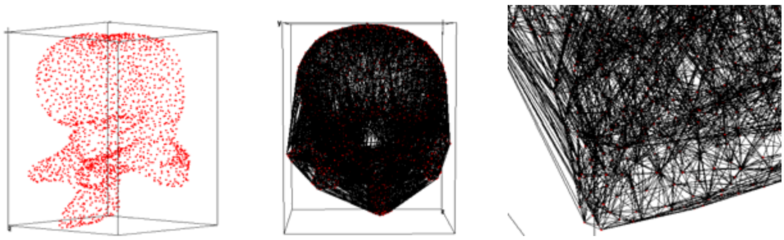
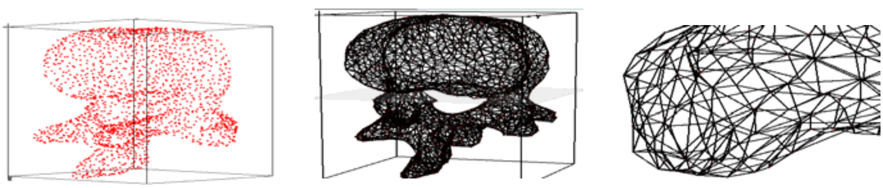


Fig. 5. Obtaining of point-cloud bounding box



(a) Original point cloud (b) Subdivision mesh (c) Local tetrahedral mesh

Fig. 6. model of animal spine



(a) Original point cloud (b) Subdivision mesh (c) tetrahedral mesh

Fig. 7. Example of point cloud subdivision algorithm

In order to verify the improvement effect of such algorithm in the paper, the step of removing peripheral tetrahedron shall be omitted firstly when conducting the experiment. Only the initial tetrahedral subdivision is carried out, and the point cloud used in volume computing test is shown in Fig. 6 (model of animal spine, including 1932 points). The tetrahedral mesh generated by scattered point cloud is on the right.

The experiment results show that the tetrahedral subdivision is to divide the convex hull only. After forming a tetrahedral mesh, many tetrahedrons are redundant data. After removing the peripheral tetrahedron according to above algorithm, the mesh image obtained is shown in Fig. 7.

All the Delaunay tetrahedrons are aggregated to form a topological structure of point cloud. Computing the volume of all tetrahedrons, namely the volume of the area included in point cloud.

### *4.3. Result analysis*

Compute the volume with three sets of point cloud and the computing results are shown in Table 2. Such results show that the computing accuracy is up to 95%. Basing on the principles of fringe projection system, the algorithm complexity is close to $O(n \lg n)$ when the point cloud is the surface points of object. Rebuild the point cloud volume of computing space on the basis of above space mesh algorithm, and compare with the traditional rebuilding method of Poisson. Poisson is to rebuild the mesh surface according to the Poisson equation solving, them estimate the indicator parameters and contour plane of mesh model by solving the solution of Poisson equation, and rebuild by using the unseamed triangle approaching to real surface. Poisson is able to create a smooth surface. Respectively rebuild the mesh by Poisson and the algorithm used in the paper. Comparing the time-consuming of those two kinds of algorithm, as shown in Table 2, for the three sets of point cloud in experiment, the time consumed of the algorithm used in the paper is 21.2%~43.9% of the time consumed by Pisson.

Table 2. Computing result of point cloud volume

| No. | $V_o/mm^3$ | $V_/mm^3$ | $\delta/\%$ |
|---|---|---|---|
| 1 | 27952.8 | 26384.648 | 5.07 |
| 2 | 58990.5 | 61591.981 | 4.41 |
| 3 | 36885.5 | 36173.610 | 1.93 |

Table 3. Comparison of algorithm time-consuming

| Point cloud in experiment | Time-consuming of Poisson method/s | Algorithm used in the paper/s |
|---|---|---|
| 1 | 3.259 | 0.691 |
| 2 | 4.403 | 1.933 |
| 3 | 80.402 | 30.553 |

## 5. Conclusion

Aiming at the excessive packaging detection of goods, a commodity packaging detection system based on grating projection measurement is proposed in this paper. Such system adopts a new measurement method computing the bounding box volume of object's point cloud instead of the circumscribed rectangular parallelepiped volume of object, and proposes a method to compute the point cloud volume of 3-D object in packaging detection, which has increased the plot ratio, realized noncontact and rapid packaging detection. The method to measure circumscribed rectangular parallelepiped by bounding box instead of manual measurement will provide more reference bases for future packaging detection. The measuring time can be obviously reduced by taking the advantage of contactless and excellent accuracy of machine vision, so it has a higher practicability.

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